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No of Questions - 07

SCHOOL OF ACCOUNTING AND BUSINESS
BSc. (APPLIED ACCOUNTING) GENERAL / SPECIAL DEGREE PROGRAMME 2014/15

YEAR I SEMESTER I (Group A)
END SEMESTER EXAMINATION – JUNE 2014

QMT 10130 Business Mathematics

Date	:	30th June 2014
Time	:	9.00 a.m. - 12.00 p.m.
Duration	:	Three (03) Hours

Instructions to Candidates:

- Answer any five (05) questions.
- The total marks for the paper is 100.
- All questions carry equal marks.
- Use of scientific calculator is allowed.
- Formula Sheet is provided.
- Answers should be written neatly and legibly.

Question No. 01

i. Show that the marginal cost [MC] must be equal to the marginal revenue [MR] at the profit maximizing level of output.

ii. Total Revenue [TR] and the Total Cost [TC] functions of a firm are

$$TR = 8400Q - 36Q^2 \text{ and}$$

$$TC = 9000 + 480Q \text{ respectively.}$$

a. Set up the profit function π .
b. Find the critical value/s where π is at a relative extremum
c. Use the second order condition to distinguish the critical point/s.
d. Calculate the maximum profit.

(Total 20 Marks)

Question No. 02

The profit function $P(x, y) = 360x + 204y - 24xy - 36x^2 - 18y^2 + 2000$ of a firm is assumed to have a monopoly on x and y .

i. Find the two first order partial derivatives P_x and P_y .

ii. Set P_x and P_y equal to zero and solve for x and y .

iii. Find the second order partial Derivatives

$$P_{xx,}\ P_{yy} \ and\ P_{xy}$$

iv. Evaluate the second order partial derivatives at the critical points obtained in part (ii).

v. Show that the following condition holds at the critical point.

$$P_{xx}\ P_{yy} > \left[P_{xy}\right]^2$$

vi. Since the above condition is satisfied, the critical point is an extremum point for the profit. Find the extremum profit?

(Total 20 Marks)

Question No. 03

i. If $U(x,y,z) = 3x^2yz + 4xy^2z + 5y^4$, without using Euler's theorem prove that

$$x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} + z\frac{\partial U}{\partial z} = 4U$$

ii. Let $U(x,y)$ be a multivariable function in x and y. U is said to be homogeneous function of order ' n ' if $U(\lambda x.\lambda y) = \lambda^n U(x,y)$.

If $U(x,y) = \dfrac{x^5 + y^5}{x^2 + y^2}$ show that $U(x,y)$ is a homogeneous function of degree three.

iii. A company has two factories that produce T.V. sets. The two factories are located at A and B. The number of units of T.V. sets produced per month by the factory located at A is $\boldsymbol{x}$ while the number of units of T.V. sets produced per month by the factory located at B is $\boldsymbol{y}$. The joint cost function for the production of T.V. sets per month is given by

$$\boldsymbol{C(x,y) = 6x^2 + 12y^2}$$

If the company has a demand of 90 units of T.V. sets per month. Find the number of units of T.V. sets that should be produced per month by each factory to minimize the cost of production per month and find the optimal cost.

(Total 20 Marks)

Question No. 04

i If $A = \begin{pmatrix} -1 & 2 \\ 3 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 0 \\ 2 & 3 \end{pmatrix}$

Find the matrix X which satisfies the following relationship $3A - B + 4X = 0$

ii Two types of radio valves A, B are available for assembling two types of radios P and Q in a small factory. The factory uses 2 valves of type A and 3 valves of type B for a type of radio P, and for a type of radio Q it uses 3 valves of type A and 4 valves of type

B. The number of valves of type A and B used by the factories are 130 and 180 respectively.

a. Identify the unknowns to be evaluated in the above problem.
b. Develop the system of simultaneous equations which represent the above problem.
c. Find the number of radios assembled through the solution of the system of simultaneous equation you developed in part (b) using the matrix method.

(Total 20 Marks)

<u>Question No. 05</u>

i. Marginal revenue of a firm is given by $MR = \frac{d}{dQ}\{TR\} = 5000 + 50Q - Q^2$.

a. Find the Total revenue function.
b. Find the number of items to be sold to maximize the revenue.
c. Find the maximum revenue.

ii. The rate of net investment is $I = 120\ t^{1/5}$ and capital stock at $t = 0$ is 150. If the relationship between capital function K and the net investment is given by $K = \int I\, dt$. Find the capital function K, using the given boundary condition at $t = 0$, $K = 150$.

iii. a. Using your knowledge on partial fractions show that

$$\frac{1}{x^2 - 4} = \frac{1/4}{x-2} + \frac{-1/4}{x+2}.$$

b. Hence or otherwise find

$$\int \frac{1}{x^2 - 1} dx.$$

(Total 20 Marks)

Question No. 06

i. A sum of £ 50000 was deposited in a bank at an interest rate of 13% compounded quarterly. Seven years later the rate decreased to 7% compounded semiannually. If the money was not withdrawn, how much was in the account at the end of 10 years after the deposit was made?

ii. Over 5 years a bond costing $ 2000 increases in value to $ 2700. Find the effective annual rate.

iii. A machine depreciates by 20 percent in the first year, then by 10 per cent per annum for the next 5 years and by 2 per cent per annum thereafter. Find its value after 7 years if its initial price is £ 720,000

iv. A company purchase a machine $ 12,000. The machine Contribute $ 3,500 per annum for five years. After 5 years it is scrapped for $ 1000. Find the Net Present Value of the machine if the interest is 5% per annum

(Total 20 Marks)

Question No. 07

i. A Manager expects the following cash flow pattern in a new project that they plan to launch. The manager needs your help to find the internal rate of return of the project

Time	Cash flow ($'000)
0	(80)
1	40
2	30
3	20
4	05

ii. A $ 60000 mortgage is taken out on a property at a rate of 10 percent for 30 years.

a. What will the monthly repayment be?

b. After 15 years of the mortgage, the interest rate increases to 13 percent, by what amount the monthly repayment figure increase.

(Total 20 Marks)

FORMULA SHEET – QMT 10130

$$V = P(1 + r\,n) \qquad V = P(1 + r)^n$$

$$V = P(1 - r)^n$$

$$P_{ODI} = R\left\{\frac{1 - (1 + r)^{-n}}{r}\right\} \qquad A_{ODI} = R\left\{\frac{(1 + r)^n - 1}{r}\right\}$$

$$P_{PER} = R\left\{\frac{1}{r}\right\}$$

$$IRR = r_1 + \left\{\frac{NPV_1}{NPV_1 - NPV_2}\right\}(r_2 - r_1)$$

$$A^{-1} = \left(\frac{1}{|A|}\right) adj(A)$$

$$AX = b \Rightarrow X = A^{-1}b$$